Crawford, Shane and Murray, Martin and Powell, John (2001) Development of a Mechanistic Model for the Determination of Track Modulus. In *Proceedings 7th International Heavy Haul Conference*, Brisbane, Australia.

Development of a Mechanistic Model for the Determination of Track Modulus

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Summary: The current world-wide trend towards increased axle loads and faster trains has resulted in increased damage to heavy haul routes. A simple, accurate method is required for determining track modulus, to improve track design and prediction of degradation. The paper describes the on-going development of a mechanistic model for track modulus. It also describes a series of field tests, with the following outcomes:

- comparison of several methods for determining track modulus based on track deflection under load,
- achieving a better understanding of track behaviour under load, and
- calibration of the mechanistic model against test results.

Index Terms: Track modulus, track maintenance, track stiffness.

1.0 INTRODUCTION

There is currently a world-wide trend towards increased axle loads and train speeds, especially on heavy haul lines. Due to the increased pressures being placed on rail track, it has become important to be able to quickly and accurately assess the structural condition of the track. In addition, it is also critical to understand track behaviour during the design phase. An over-designed track leads to overspending on infrastructure, while an under-designed track will tend to deteriorate at a rapid rate, significantly increasing maintenance costs.

Some researchers have proposed that measurement of track deflection under an applied vertical load may be used to assess the structural conditions (especially stiffness) of the track, the results being expressed as track modulus. Track modulus is generally considered to be an important parameter, although it is seldom measured and its magnitudes are at best approximately known for most sections of railway track (Selig and Li, 1994).

A number of theoretical models have been advanced for the calculation of track modulus based on load vs deflection relationships, yet there is no consensus on the best or most accurate method. The most commonly known method is known as the beam-on-elastic-foundation method, or the Winkler method. A vertical force (P) applied by a wheel produces a vertical rail deflection (w). Therefore, the track stiffness (k), taken at a point as the wheel passes directly overhead, is defined as;

$$k = \frac{P}{w}$$

From this, the track modulus (u) is defined as;

$$u = \frac{(k)^{4/3}}{(64EI)^{1/3}}$$

where E is Young's modulus of the rail steel and I is the rail moment of inertia.

While this model is theoretically accurate, it has a number of practical shortcomings

- The method does not take into account the initial closing of voids in the track upon application of the load. Known as the 'seating' modulus, it is believed that the method therefore does not give an accurate representation of structural capacity of the track
- By considering only a selected point of the track, the values obtained are subject to variation due to local inconsistencies at the test site. Taking a reading at a single point may not representative of the entire track
- Unless using specially modified testing equipment, the proximity and load of other axles will have an

influence on the deflection measured at the test location. Failure to consider other loads will have an impact on the accuracy of the results obtained.

Although the Winkler method can be modified to take into account non-linearity of the track (Selig and Li, 1994), this still leaves the issues of localised site differences and surrounding axles. Other methods have attempted to resolve these issues.

The deflection basin method is based on the vertical equilibrium of forces acting on the rail. This relationship may be expressed as

$$\sum P = uA$$

where P is the wheel load, u is the track modulus and A is the area of the deflection basin (ie. the area between the original rail position and the deflected rail position). This method effectively handles the issues of localised site differences and nearby axles, and may be modified as follows to take into account non-linear behaviour of the track

$$\sum (P_f - P_o) = u(A_f - A_o)$$

where P_f and A_f are the final loads / bowl areas, and P_o and A_o are the loads and bowl areas at which the track is considered to be fully 'seated' (ie. all voids are closed and the track is behaving in its full structural capacity).While this method is considered to give the best reflection of track structural capacity, it is extremely time-consuming to take the numerous deflections required for the calculations.

A simpler method proposed by Kerr (1983) considered the effect of multiple axles on a single point deflection. The intention was to provide a method by which track modulus measurements may be made in the field without the use of specialist loading devices. For a standard 6-axle locomotive, the following equation may be used to determine the track modulus (k);

$$\frac{w_m}{P} = \frac{\beta}{2k}[1 + \sum_{i=1}^{5} n_i e^{-\beta l_i} (\cos \beta l_i + \sin \beta l_i)]$$

The values l_1 to l_5 are the axle spacings of the locomotive (refer Figure 1.1), and the values n_1 to n_5 are factors based on the ratio of the following axle loads as compared to the first axle. They are determined by taking the first axle load (P), and comparing the following axles as such;

$P_0 = P$	$P_1 = n_1P$	$P_2 = n_2P$
$P_3 = n_3P$	$P_4 = n_4P$	$P_5 = n_5P$

w_m is the deflection measured beneath the first axle, and β is determined as follows;

$$\beta = \sqrt[4]{\frac{k}{4EI}}$$

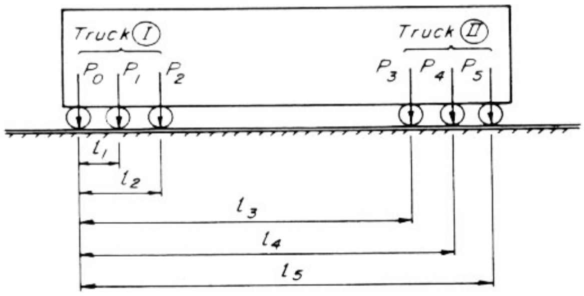

Figure 1.1 – Layout of axle spacings for Kerr's method

While all of these methods have various advantages and disadvantages, none of them are of any use in the determination of track modulus for a proposed section during design. In addition, while the field measurements are believed to provide reasonable values for track modulus, they do not enable those performing the testing to distinguish the factors causing them, nor how to repair or adjust the track in order to alter these values should they not prove to be satisfactory. For this reason, a mechanistic model is required, so that track modulus readings may be utilised to give a meaningful assessment of the track and of its components.

In response to these needs, some researchers have developed computer models (eg. GEOTRACK, Chang et al, 1980) for analysis of the behaviour of the track and foundation material, but these are not necessarily readily available. The model described in this paper was originally developed at Queens university in Canada (Cai *et al*, 1994), and has since been modified to suit a wider variety of conditions (Zhang *et al*, 1998). It considers sleeper bending rigidity together with elastic properties of the railpad and the layered ballast/subballast/formation, and is based on the assumption that the sleeper acts as a flexible beam resting on an elastic medium. The parameters considered in the model are the combined vertical stiffness of the sleeper and railpad, sleeper spacing, and the equivalent spring stiffness of a sleeper lying on the track foundation; this equivalent spring stiffness embodies ballast, subballast and formation elastic properties.

2.0 TESTING REGIME

2.1 General description of sites

Test sites on rail tracks near Rockhampton in Central Queensland were selected for this project for the following reasons;

- Both heavy haul (coal) lines and freight lines were within close proximity to the Rockhampton workshops and marshalling yard.
- Coal mines using some of the lines have a regular ‘shut-down’ for maintenance purposes one day per week, allowing access to the coal haul track without interrupting scheduled vehicles.
- The branch line between Rockhampton and Yeppoon had a passing loop, which could also be accessed without interrupting regular services.

Conditions in Rockhampton had been dry for some months prior to testing. This was also indicated in the moisture content of the soil samples which were taken from each site (ranging from 2.7-5.5%).

2.2 Sites One and Two

These two sites were situated on the North Coast heavy haul coal line, between Bajool and Archer in Central Queensland. Both sites consisted of 60kg/m rail on prestressed concrete sleepers. The sites were chosen to give an indication of stiffness variations from formations with clean ballast and heavily fouled ballast. A cross-section of Site 1 is shown in Figure 2.1.

The ballast at Site 1 was extremely fouled with coal dust below the base of the sleeper. The state of the ballast and the formation can be seen in Table 3.2.

Site Two was located a few kilometres south from Site 1, along the same line The ballast at this site had been recently replaced, and was quite clean. See Table 3.2 for the ballast and formation descriptions. The cross-section at Site 2 was similar to that shown in Fig.2-1.

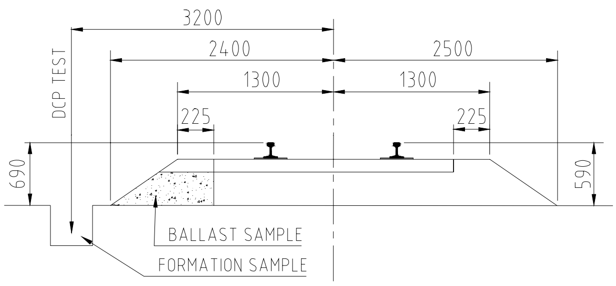

Figure 2.1 – Track structure cross-sections; site one

2.3 Sites Three and Four

These sites were situated on the Yeppoon branch line, at the Lakes Creek Siding. Both sites consisted of 41kg/m rail, Sites Three and Four having steel and timber sleepers respectively. These sites were chosen to give indications of stiffness variations from tracks with different sleeper types. A cross section of Site 3 is shown in Figure 2.2.

The ballast at Site 3 had been renewed approximately 12 months earlier, and was found to be well within the allowable ranges for Grading B ballast. During the ballast sampling, it was noted that the ballast was heavily fouled beyond 150mm below the sleeper. See Table 3.2 for further ballast and formation details.

Site 4 was located 400m along the same line from Site 3, but with timber sleepers. The ballast at this site was in good condition, but was found to be of a very low depth (100mm below the sleepers). Analysis of a ballast sample taken from the site found that a high percentage of the sample passed the lower sieve sizes, indicating degraded ballast. Further details are in Table 3-2

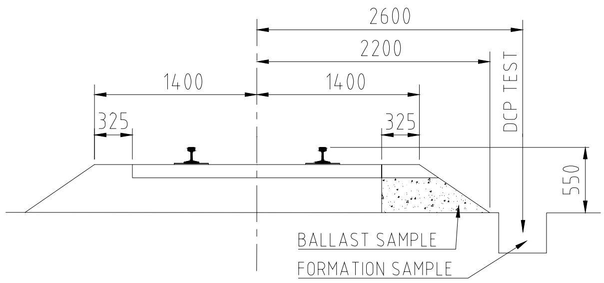

Figure 2.2 – Track structure cross-sections; site three

2.4 Testing apparatus

The test train was made up of three rail vehicles:

- A 90 tonne 6-axle locomotive, used to move the vehicles from one site to another,
- An intermediate wagon, which served as a buffer between the locomotive and the test wagon, to reduce the transmission of vibrations from the loco to the test wagon.
- The test wagon. This vehicle was a freight wagon which was currently under load with flood rock for emergency repair work. The gross weight of the wagon was 57 tonnes, or 14.3t per axle. This vehicle was modified to enable transducers and hydraulic jacks to be attached. Refer to Figure 2.3 for a photograph of the test wagon.

Figure 2.3 – Test wagon

2.5 Description of testing

Two different tests were carried out at each of the selected sites. The first of these was conducted while the locomotive was moving the test wagon into position. One "independent" arm was set up either side of the track and attached to one of the rails; displacement transducers monitored vertical displacement of each rail as the vehicles moved into position.

Once the test wagon was in the desired position, two hydraulic jacks were attached to the underside of the wagon to apply a single point load to each rail. In addition to the independent arms, a series of 22 transducers were attached to the wagon, to measure the vertical deflection of the rail caused by the load applied by the jacks. From these measurements the area of the deflection basin will be able to be determined, though the results of these area calculations will be reported in a further publication.

Two tests were conducted at each site, the second taking place approximately three sleeper spacings beyond the first test site. This was done in order to ascertain the repeatability of such tests, and to obtain some idea of the variability of track modulus over a site with identical superstructure characteristics and very similar substructure characteristics.

2.6 Aims of testing

- To obtain track modulus values for a variety of track structures, in order to compare the test values to those obtained by the mechanistic model, for calibration purposes.
- To compare with the empirical values currently used by Queensland Rail for track design purposes.
- To calculate track modulus values from the recorded data using various methods, in order to compare the accuracy of each method.

3.0 OUTCOMES

3.1 Extraction of modulus from data.

Table 3.1 shows the values of track modulus extracted from the track deformation data measured at the four sites under different loadings.

Various columns in the table represent five different methods by which track modulus was calculated at the four sites. How were these values determined?

The Winkler method provides a means of calculating modulus from the deflection at a point in the track. At each of the four test sites, the mean peak deflection of the track under each loco axle was used together with 147kN (=15t) axle load to produce the moduli in the 3rd column of Table 3.1. The same process was used with the deflections of the test wagon together with an axle load of 140kN (=14.3t) to determine the moduli listed in the 4th column of Table 3.1.

Using the Winkler method also enabled extraction of track modulus from the data of load/deflection measured at the hydraulic jacks (see Fig. 3.1). The deflection experienced at a total jack force of 147kN at each site was used to give the moduli in the 5th column of Table 3.1.

An alternative method of determining track modulus is that given by Kerr (1983) as described earlier in Section 1.0; the moduli so determined are listed in the 6th and 7th columns of Table 3.1.

Then, in the 8th column of Table 3.1, the mean results listed are the average for each site of the moduli determined by the five methods.

3.2 Comparison of sites and methods

(a) Methods.

The determination of track modulus has been fraught with difficulty and prone to error – see for example Scott *et al* (1982). However, Table 3.1 shows that the moduli determined with the various methods do not vary excessively amongst the methods used. Future publications will enable a comparison between these methods, which rely on point deflections, with those that use the full deflection bowl to determine modulus.

Table 3.1 Track Modulus Calculations by Various Methods (MPa)

Site No.	Ballast & sleeper descri-ption	Point deflection			Multi-axle deflection		Mean of test data results	Values reported by others for similar sites			
		Loco	Test Wagon	Test Jack	Loco	Test Wagon		Murray Griffin 1993	West-rail 1994	Eber-sohn 1994	Stew-art 1985
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)				
1	Fouled concr.	22	21	26	26	30	25	-	-	-	-
2	Clean concr.	58	52	78	69	72	66	-	-	57	52
3	Clean steel	15	15	- *	16	18	16	22	-	-	-
4	Clean timber	12	12	16	13	15	13	13	14	-	15

* wrongly positioned transducer gave meaningless data at this site.

(b) Sites.

A comparison between sites shows that the moduli for sites 2, 3 and 4 in Table 3.1 are within expectations:

- Site 2 is a heavy haul line with clean ballast, and concrete sleepers; its ballast and formation properties (see Table 3.2) are what one would expect for such a site. The mean modulus of 62MPa compares reasonably well with the Ebersohn and Stewart values of 57MPa and 52MPa respectively for similar sites; the test value is probably on the high side because the formation at Site 2 had a low moisture content due to a prolonged dry period, leading to a higher modulus of elasticity in the formation. Nevertheless, these are all much higher than the 25-30MPa modulus recommended for design of heavy haul track (Hagaman, 1995).
- Site 3 is a branch line for typical mixed traffic, with steel sleepers, clean small sized ballast and a reasonable formation. A modulus of 15MPa is on the low side for such track (eg in comparison with the Griffin value of 22MPa); however the small size of the ballast material may have contributed through lowering the value of the ballast's modulus.
- Site 4 is further along the same branch line, and is a mature track with timber sleepers, clean ballast, and reasonable formation. The value of 13MPa in Table 3.1 is a fair value for such a site and compares well with the values reported in the other three studies.

However, the modulus for Site 1 is quite low, about 40% of that at Site 2. This site is on the same heavy haul line as Site 2 and has the same track structure and ballast stone. The formation at Site 2 is composed roughly 50% of low plasticity silt and clay (suggesting a low formation elastic modulus), though it has a high CBR (see Table 3.2). The only other difference between the two sites is that Site 1 has ballast that is heavily fouled with 30% coal dust by volume. It's probable that the low track modulus is due to both a high fines content in the formation together with the coal dust acting as a lubricant (like graphite powder) between the ballast stones, enabling much greater movement within the ballast mass.

Now, it is well known that the presence of fouling material such as coal dust or worn ballast fines can have a drastic effect on the performance of track during wet weather. Such fouling material tends to hold water and prevent free draining of the ballast, leading to softening of the formation and increasing track roughness. However, it's clear from comparing Sites 1 and 2 that large amounts of fouling material can also drastically affect the track's dry performance.

3.3 Variability of modulus with load

Those designing track use a value of modulus that is assumed to be dependent solely upon track structure parameters, and that this structure is linear elastic.

Kerr and Shenton (1986) discussed what they claimed to be a bi-linear relationship between vertical load and deflection on a track, that reflects a soft settling in phase, followed by a stiffer "contact" phase.

Fig.3.1 is a plot of the vertical displacement of the track at Site 2 versus the load applied by the test jacks. The shape of the graph is typical of all four sites. The graph shows a continuous non-linear behaviour rather than bi-linear. The transfer of force through ballast is by point contact between ballast stones; as the magnitude of these forces increases, deformation of the stones leads to increased contact area between the stones, steadily increasing the apparent stiffness of the ballast mass.

Fig.3.1: Typical load/deflection plot from test jacks

The absence of linearity in the force/displacement behaviour of the track means that there is no unique modulus able to be determined for a track. Fig.3.1 also shows three separate straight lines for which three different moduli may be calculated, depending upon how the graph is interpreted. Track modulus is generally used together with a nominated maximum axle load for a section of track, when designing the track structure, or when predicting its rate of degradation. So, the logical value of force to be used in extracting a track modulus from data such as is shown in Fig.3.1, is the nominated maximum axle load for the track. Therefore, the deformation implied in using such a modulus would be representative of the actual deformation that would occur in that track under that nominated maximum load.

The values of modulus determined in Table 3.1 for this test program used a track deflection corresponding to an axle load of 15 tonne for ease of comparison; 15t was roughly the axle load for the test loco and for the test wagon. The solid straight line in Fig.3.1 represents this situation, giving the 78MPa test jack ("secant") modulus in Table 3.1. Seeing as Site 2 is a heavy haul line with an actual nominated maximum axle load of 26t, then a more appropriate secant modulus would in fact be 158MPa rather than 78MPa, as shown by the dashed line in Fig.3.1.

What if this rail line is being considered for increased axle loading? In Fig.3.1 the tangent or "contact" modulus for

Site 2 at a load of 26t is >300MPa, as shown by the dotted line. This is very much greater than any modulus used for the original design of the track. In other words, tracks are much stiffer than the engineer might think in circumstances of possible increased axle loads. However, this does not take into account the greatly increased degradation and wear that the track would experience due to such an increase in load. There are comprehensive models available that can predict this degradation (Zhang *et al* 1998).

3.4 Comparison with Track Modulus model

The mechanistic track modulus model described in Section 1.0 has been used with success to anticipate the modulus for typical lines around the state of Queensland (Zhang *et al*, 1998). A sensitivity analysis using this model has shown clearly that the modulus of elasticity of the formation material is the dominant factor in determining track modulus; this has been confirmed by studies using the well known GEOTRACK model (Selig and Li, 1994). The thickness of the ballast has an important but relatively small contribution to track modulus.

As reported in Section 2, the properties of samples of both the ballast and formation material were determined for the four test sites and are shown in the 2nd , 3rd and 4th columns of Table 3.2. On the basis of the standard geotechnical descriptions given of the formation material, the elastic moduli in the 5th and 6th columns of Table 3.2 were deduced.

The great variability in the moduli deduced from the CBR's at each site suggests that this approach is not reliable, though it is used widely to predict the modulus for road pavements. So, using the other values of elastic modulus in Table 3.2 in the model gives the track moduli listed in the 2nd column of Table 3.3.

Clearly the predicted and measured moduli (2nd & 3rd columns Table 3.3) do not correspond. Now, the model predicts the contact modulus, with no regard for any settling in phase. The measured contact moduli for the four sites are shown in the 4th column of Table 3.3. It was found from the load/displacement curves that this contact modulus at 15t load was about twice the normally used secant modulus for 15t. Consequently, the predicted secant moduli in the 5th column of Table 3.3 are just half the values in the 2nd column of the Table.

The Table shows that these final moduli are reasonably close to the measured values for the heavy haul Sites 1 and 2, but are twice as big as those measured at Sites 3 and 4. Additional development of the model seems to be warranted for lightly trafficked lines.

Table 3.2 Ballast and Formation Properties

Site (1)	Ballast description (2)	Geotechnical description of formation (3)	Formation components by weight [1] (4)	Form.Matl. Modulus [2] (5)	Form.CBR Modulus [3] (6)
1	Heavily fouled (18% coal), 8% >37.5mm sieve, 37% < 19mm	GC-clayey gravel, medium plasticity, clayey medium gravel sand mix; CBR=13	20% gravel, 34% sand, 46% silt & clay	44MPa	130MPa
2	A grade, clean, 55% >37.5mm sieve, 1% <19mm.	GC-clayey gravel, medium plastiticy, well graded medium-coarse gravel, pale brown mottled grey; CBR=N/A	58% gravel, 25% sand, 17% silt & clay	98MPa	N/A
3	B grade, 46% >37.5mm sieve, 14% < 19mm.	GW-GM well graded gravel-silty gravel, low plasticity, well graded sandy gravel with silt; CBR=20	61% gravel, 31% sand, 8% silt & clay	102MPa	200MPa
4	B grade, 4% > 37.5mm sieve, 14% < 19mm.	GW-GM well graded gravel-silty gravel, low plasticity, well graded medium-coarse gravel with silt; CBR=7	55% gravel, 38% sand, 7% silt & clay	95MPa	70MPa

[1] gravel is retained on a 2.36mm sieve; silt & clay pass a .075mm sieve.
[2] formation elastic modulus deduced on the basis of typical elastic moduli for the components (see Zhang, 1999) and with regard to the proportions of materials in a composite (John, 1983).
[3] formation elastic modulus =10 x CBR as prescribed in NAASRA (1992).

Table 3.3 Predicted and Measured Track Moduli

Site (1)	Predicted Track Modulus using column 4 from Table 3.2 (2)	Measured Track "secant" Modulus (from Table 3.1) (3)	Measured Contact or "Tangent" Track Modulus (dotted line in Fig.3.1) (4)	Predicted Track "secant" modulus (0.5 times column 2) (5)
1	58MPa	24MPa	45MPa	29MPa
2	102MPa	62MPa	107MPa	51MPa
3	67MPa	15MPa	30MPa	33MPa
4	53MPa	13MPa	28MPa	27MPa

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4.0 CONCLUSIONS

- Track moduli measured at different sites in Central Queensland were much the same as values reported in different parts of the world for sites with characteristics similar to the present sites.
- Measured moduli for low axle load track were similar to recommended design values, but were very much higher than values used to design heavy haul track.
- Heavy fouling by coal dust will not only cause retention of water and soften formation during wet periods, but may also "lubricate" the contacts between ballast stones, giving a softer track even in fully dry conditions.
- Determining track modulus from displacement of a point in the track gives very reasonable results.
- The continuously curved load-displacement relationship measured at the four sites shows that track modulus to be used in design is dependent upon the design axle load being considered.
- This non-linearity also implies that track is much stiffer than a design modulus implies, when considering use of increased axle loads on an existing heavy haul line
- A model under development for use in predicting track modulus gave good results for the heavy haul line sites, but poorer results for lighter track sites.

5.0 REFERENCES

1. Selig, E.T., and Li, D. **Track Modulus: It's Meaning and Factors Influencing It.** Transportation Research Record 1470, TRB, National Research Council, Washington, D.C., 1994, pp. 47-54.
2. Ebersohn, W., and Selig, E.T. **Track Modulus Measurements on a Heavy Haul Line.** Transportation Research Record 1470, TRB, National Research Council, Washinton, D.C., 1994, pp. 73-83
3. Cai, Z., Raymond, P., and Bathurst, R.J. **Estimate of Static Track Modulus Using Elastic Foundation Models.** Transportation Research Record 1470, TRB, National Research Council, Washington, D.C., 1994, pp. 65-72
4. Zhang, Y., Murray, M., and Ferreira, L. **A mechanistic approach for estimation of track modulus.** Conference on Railway Engineering, Rockhampton, Australia, September 7-9, 1998, pp. 9-14.
5. Kerr, A.D. **A Method for Determining The Track Modulus Using a Locomotive Or Car On Multi-Axle Trucks.** Proc. AREA, Bulletin 692, Vol. 84, 1983, pp. 269-279
6. Kerr, A.D., and Shenton, H.W. **Railroad Track Analyses and Determination of Parameters.** Journal of Engineering Mechanics, Vol. 112, No. 11, 1986, pp. 1117-1134.
7. Chang, C.S., Adegoke, C.W. and Selig, E.T. **Geotrack model for railroad track performance.** Journal of the Geotechnical Engineering Division, ASCE, Vol.120, No.6, 1980, pp.939-957.
8. Stewart, H.E., **Measurement and Prediction of Vertical Track Modulus.** Transportation Research Record 1022, TRB, National Research Council, Washington, D.C., 1985, pp. 65-71.
9. Barrett-Lennard, I., **Railways of Australia vehicle/track studies: Study No. 2 : Volume One draft executive summary & recommendations.** Railways of Australia (ROA), Westrail Civil Engineering Branch, 1994.
10. Hagaman, B.R. **Track Design.** Railway Civil Engineering Course 1995, Queensland Rail, Brisbane
11. Scott, J.F., and Charenko, A. **The Measurement of Track Modulus.** Track Structures Test Facility Progress Report No. 5, CN Rail Research Centre, St. Laurent, Quebec, January 1982
12. John, V.B. **Introduction to Engineering Materials**. MacMillan, London, 1985.
13. NAASRA. **Pavement Design.** Publication AP-17/92, Austroads, Sydney, 1992.
14. Murray, M.H. and Griffin T. **In-service Displacements of Steel and Timber Sleepers.** Proceedings, 13th ACMSM conference, University of Wollongong, 5-7 July, 1993, pp 619-626.
15. Zhang Y.J. **An Integrated Rail Track Degradation Model.** PhD thesis, Queensland University of Technology, Brisbane, 1999.